EOC Math I Remediation Lessons

Nevada Department of Education

Christina’s Rabbit Pen: Creating and Interpreting Quadratic Functions

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Students will create and interpret quadratic equations. Students should interpret the key features of the quadratic function and how they relate to real world applications.

Launch

Christina wants to build a rabbit pen so her rabbit will have space to move around safely. Christina purchased a 72-foot roll of fencing to build a rectangular pen.

- If Christina wants to build a pen with the largest possible area, what dimensions should she use for the sides? Justify your answer.
- Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.
- Refer to the “Investigate” section below for possible student approaches and questions to further student understanding.

Understand the Problem

- Are there any words you don't understand?
- What are you asked to find?
- Is there enough information to find a solution?
- Can you restate the problem in your own words?
- Or, what information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice students will build skill in choosing efficient strategies.
- Do not validate/invalidate any strategies, but ensure that students have a place to start (even if you know it will not work).
- Do not force your plan/reasoning on students.

Investigate

- Let students engage in a productive struggle.
- Monitor as students work.
- Do not offer feedback.
- Only ask questions.
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Students as they Work

(If you observe _____, then you might ask _____.)

If students do not have a starting point, then ask:

- What are some ways you might use a table for this situation?
- How could you use a diagram of the situation?

If students use 72 as the area (*xy* = 72), then ask:

- What are you using the 72 feet of fencing for in your equation?
- What does the 72 represent?

If students divide by four, then ask:

- Did you check other solutions?
- How did you get this answer?
- Why did you divide by 4?
- Will this work every time?
- Where is the four in your equation?

If students struggle with trial and error, then ask:

- How are you organizing your trials?
- What do your values represent?

If students solve by taking the square foot of 72 or 36, then ask:

- Can you explain why you are taking the square root?
- What will this value represent?

If students do not realize that this is a quadratic model, then ask:

- What kind of graph has a maximum value?
- Does a linear graph have a maximum?

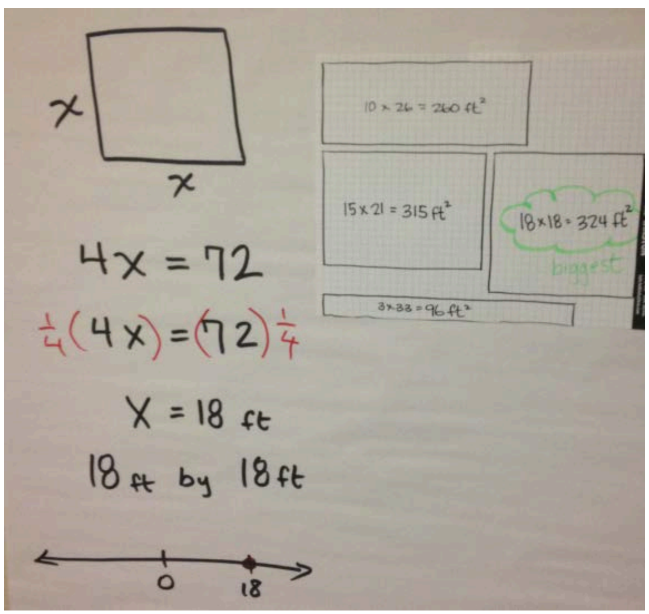
Debrief

Whole or Small Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar format.
- Have students sequence multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, model your own questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide purposeful feedback and ask questions.

Sample Solution 1

(If you observe _____, then you might ask _____.)

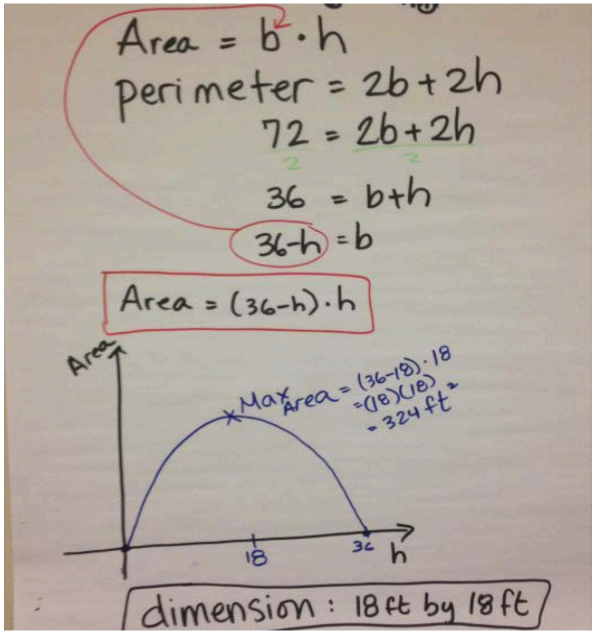


This is a common incorrect response. You might ask:

- I see that you tried four different rectangles, are those all of the possibilities?
- What do the parts of your equation represent?
- What does the 4*x* represent?
- What does the 72 represent?
- Did you check other solutions?
- How did you get this answer?
- Why did you multiply by one-fourth?
- Will this work every time?
- The problem asks for both an equation and a graph, do you have both?
- Can you show this on the coordinate grid?

Sample Solution 2

(If you observe _____, then you might ask _____.)

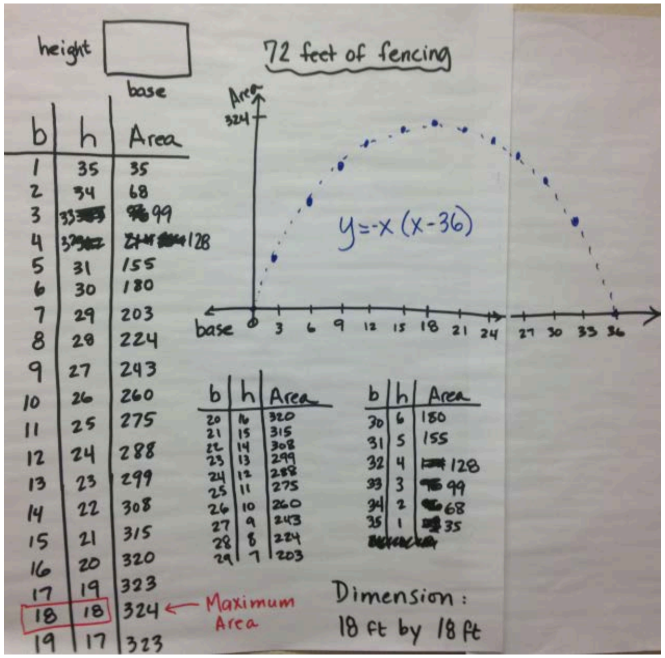


This is one possible correct response. You might ask:

- How did you get this answer?
- The problem asks for both an equation and a graph, do you have both?
- How does your graph help you determine the dimensions of the pen?
- What does the 72 represent?
- How did you get the 36?

Sample Solution 3

(If you observe _____, then you might ask _____.)



This is one possible correct response. You might ask:

- How did you get this answer?
- The problem asks for both an equation and a graph, do you have both?
- How does your graph help you determine the dimensions of the pen?

- How did you get the 36?
- Explain how you got your equation.

Other Common Errors

- When students mistake *72* as the area of the rectangle, then they confuse perimeter and area.
- When students report the area as *324 ft2* for the first part of the question, then they do not understand the "reasonableness" of their answer and should re-read the question.
- When students write the model as *y = x(x-36)*, then they made a sign error.
- When student's visual model opens up instead of down, then they incorrectly wrote the model.
- When students think that *36* is a dimension, then they do not understand the "reasonableness" of their answer.

Synthesize and Apply

When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Monitor student work and facilitate discussions by asking questions.

Extension Problem 1

Part I: Find the area of the largest possible rectangular garden that could be surrounded by 96 feet of fencing.

Solution: Find the model to represent the situation. $A = x(48 - x)$

To find the maximum area of the quadratic model, find the y-coordinate of the vertex. $x = \frac{-b}{2a} = \frac{-48}{2(-1)} = 24$, *A* = 24(48 – 24) = 576 square feet.

Part 2: Suppose you want to use one side of the house as one of the sides of the fenced area. What is the largest area possible with the 96 feet of fencing?

Solution: Find the model to represent the situation. $A = x(96 - 2x)$

To find the maximum area of the quadratic model, find the y-coordinate of the vertex. $x = \frac{-b}{2a} = \frac{-96}{2(-2)} = 24$, A = 24(96 – 2(24)) = 1,152 square feet.

Part 3: Suppose the garden is in one corner of a walled courtyard. It only needs two sides of fencing. What is the largest area possible with the 96 feet of fencing?

Solution: Find the model to represent the situation. $A = x(96 - x)$

To find the maximum area of the quadratic model, find the y-coordinate of the vertex. $\boldsymbol{x = \frac{-b}{2a} = \frac{-96}{2(-1)} = 48}$, A = 48(96 – 48) = 2,304 square feet.

Extension Problem 2

The path of a rocket is given by the following equation: $h = -16\mathrm{t}^2 + 128t$, where *h* is in feet and *t* is in seconds.

Part I: How long is the rocket in the air?

Solution: Let *h* = 0 and solve for *t*. $0 = -16t^2 + 128t$

0 = -16*t*(*t* + 8), so *t* = 0 or *t* = 8. The rocket is in the air for 8 seconds.

Part 2: What is the greatest height the rocket reaches?

Solution: Find the y-coordinate of the vertex to find the maximum height.

$x = \frac{-b}{2a} = \frac{-128}{2(-16)} = 4$, so $h = -16(4)^2 + 128(4) = 256$ feet

Part 3: At what time does it reach its highest point?

Solution: The rocket reaches its greatest height at the x-coordinate of the vertex or 4 seconds.

Part 4: How high is the rocket after 2 seconds? Is the rocket going up or down?

Solution: Substitute *t* = 2 in the model. $h = -16(2)^2 + 128(2) = 192$ feet. Two seconds is before our maximum height at four seconds, so the rocket is going up.

Part 5: At what time will the rocket return to the height found in Part IV?

Solution: Due to the symmetry of the graph, the rocket will return to 192 feet at 6 seconds.

Extension Problem 3

A surfboard shop sells 40 surfboards per month when it charges \$500 per surfboard. Each time the shop decreases the price by \$10, it sells 1 additional surfboard per month. How much should the shop charge per surfboard to maximize the amount of money earned? What is the maximum amount the shop can earn per month? Explain.

Solution: Find the mathematical model to represent this situation.

$P = (40 + x)(500 - 10x)$, where *P* equals Profit

Placing the model in standard form, we get: $P = -10x^2 + 100x + 20000$

To find the maximum value, find the vertex of the quadratic model.

$x = \frac{-b}{2a} = \frac{-100}{2(-10)} = 5,$ so $P = -10(5)^2 + 100(5) + 20000 = 20{,}250$

The maximum amount the shop can earn per month is \$20, 250. The shop should charge \$450 per surfboard. (500 – 10(5))

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Standards

- This task might address the following standards (standards might vary based on discussion)
  - HSA.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
  - HSA,REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

- HSF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**
- HSF.IF.C.8.a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Polya, G. (2014). How to solve it: *A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Name___________________________________

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