

# **Daisy's Pennies: Using Explicit and/or Recursive Functions**

---

## **Enduring Understanding**

(Do not tell students; they must discover it for themselves.)

Represent mathematical or real world contexts using explicit and/or recursive functions. Reason quantitatively using geometric sequences.

Note: Adapted from Hendrickson et al., 2012.



## **Launch**

Daisy's parents have offered to pay her to do her chores every day. Daisy is worried that the deal she made with her parents isn't fair to her. As Daisy does her chores her parents want to pay her 1 cent on the first day, 2 cents the second day, continuing to double the amount each day that she does her chores.

1. Model the money that Daisy would get from doing her chores on any given day.
2. Predict the amount of money that Daisy would earn on the 30<sup>th</sup> day with this plan.
3. Write both a recursive and an explicit formula that shows the money that Daisy is paid on any given day of this plan.
4. On which day will Daisy receive about \$15 for doing her chores?
5. How much total money will she earn during the first week?
6. Would you take this deal? Why? Why not?

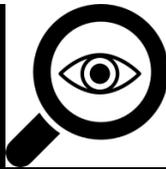
## **Understand the Problem**

- Are there any words you don't understand?

- What are you asked to find?
- Is there enough information to find a solution?
- Can you restate the problem in your own words?
- Or, what information do you need to find?

## Develop a Plan

- There are many reasonable ways to solve a problem. With practice students will build skill in choosing efficient strategies.
- Do not validate/invalidate any strategies, but ensure that students have a place to start (even if you know it will not work).
- Do not force your plan/reasoning on students.



## Investigate

- Let students engage in a productive struggle.
- Monitor as students work.
- Do not offer feedback.
- Only ask questions.
  - Why did you choose that number?
  - What assumptions did you make?
  - Explain what you are doing here.
  - What does that solution mean?

## Questions for Students as they Work

(If you observe \_\_\_\_\_, then you might ask \_\_\_\_\_.)

**If students are having a hard time getting started, then ask:**

- How might you organize your data in order to observe a pattern?
- What totals can you calculate?

- Do you think you have enough data points?
- How could you find more?

**If students have a table but no equations, then ask:**

- What patterns do you notice in your table?
- How would you describe this relationship?
- How might you describe that pattern in symbols?
- How might your table help you know if your equation is correct?

**If students are adding 1 cent daily rather than doubling each day, then ask:**

- What does doubling mean?
- Can you give some examples of doubling?
- How does this effect your reasoning?

**If students have  $f(n) = .01(2)^n$ , then ask:**

- Could you use a table to check the accuracy of your equation?

**If students have  $f(0) = .01$ ,  $f(n) = f(n - 1) \cdot 2$ , then ask:**

- What does  $f(0)$  mean in this context?
- What would  $f(1)$  mean in this context?
- What does  $n$  represent?



# Debrief

## Whole or Small Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar format.
- Have students sequence multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, model your own questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide purposeful feedback and ask questions.

## Questions to Consider

**Consider asking these questions as students examine both explicit and recursive forms of this function:**

- When might one form be more useful than another?
- What similarities or differences are present in the functions?
- Are any of the functions equivalent? How do you know?
- What type of sequence is modeled? (arithmetic, geometric)
- What type of function is modeled? (linear, exponential)
- Consider including domain, range, continuity, and other key features of functions in the discussion.

**If students' samples are in one notation, then ask:**

- Could you introduce other notations?



## Synthesize and Apply

When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Monitor student work and facilitate discussions by asking questions.

### Extension Problem 1

Daisy's brother, Isaac, has agreed to a different method of payment for doing his chores. He agrees to do his chores for \$2 for the first week, \$4 for the second week, continuing to double for the entire year.

- Model the money that Isaac would get for doing his chores during any given week.
- Predict how much money Isaac would earn in week 26.
- Write both a recursive and an explicit formula that shows the money that Isaac is paid on any given week.
- How much money will Isaac earn in the last week of the year?
- How much total money will Isaac earn in the first six weeks?
- Do you think Isaac's parents will stick with this plan? Why or why not?

### Extension Problem 2

A school has collected 100,000 pennies to donate to local charity organizations. They want to donate 60% of the pennies each day to a different charity.

- Model this situation with an explicit and recursive function.
- How many charities can they donate to before running out of money?
- How much money will the third charity receive?
- What is the difference in money received by the first and third charity organization?

### Extension Problem 3

- Find the first 3 terms of the sequence in the table below.
- Find the 30th term in the table below. Show which function form you will use to find the 30th term. Justify your choice.

$n$	$f(n)$
1	
2	
3	
4	4000
5	2000
6	1000
7	500

## References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

### Standards

This task might address the following standards (standards might vary based on discussion):

- HF.BF.A.1 Write a function that describes a relationship between two quantities.
- HF.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- HF.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*
- HSA.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Hendrickson, S., Honey, J., Kuehl, B., Lemon, T., & Sutorius, J. (2012). Secondary mathematics 1: An integrated approach. Salt Lake City, UT: Utah State Office of Education and Mathematics Vision Project.

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.

Name \_\_\_\_\_

## Daisy's Pennies

Daisy's parents have offered to pay her to do her chores every day. Daisy is worried that the deal she made with her parents isn't fair to her. As Daisy does her chores her parents want to pay her 1 cent on the first day, 2 cents the second day, continuing to double the amount each day that she does her chores.

1. Model the money that Daisy would get from doing her chores on any given day.
2. Predict the amount of money that Daisy would earn on the 30<sup>th</sup> day with this plan.
3. Write both a recursive and an explicit formula that shows the money that Daisy is paid on any given day of this plan.
4. On which day will Daisy receive about \$15 for doing her chores?
5. How much total money will she earn during the first week?
6. Would you take this deal? Why? Why not?