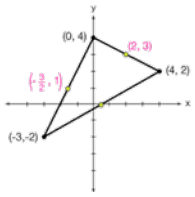


Titanic: Determining if Two Events are Independent

Enduring Understanding

(Do not tell students; they must discover it for themselves.)

Given a data set, students will be able to determine if two events are independent.

Standards

HSS-CP.A.4 **Understand independence and conditional probability and use them to interpret data.** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. *(Modeling Standard)

HSS-CP.A.5 **Understand independence and conditional probability and use them to interpret data** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. *(Modeling Standard)

Launch

Introduce the Task

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Data on survival of passengers are summarized in the table below. We will use this data to investigate the validity of such claims. (Data source)

	Survived	Did not survive	Total
First class passengers	201	123	324
Second class passengers	118	166	284
Third class passengers	181	528	709
Total passengers	500	817	1317

- Are the events "passenger survived" and "passenger was in first class" independent events? Support your answer using appropriate probability calculations.
- Are the events "passenger survived" and "passenger was in third class" independent events? Support your answer using appropriate probability calculations.
- Did all passengers aboard the Titanic have the same probability of surviving? Support your answer using appropriate probability calculations.

Understand the Problem

- Are there any word(s) you don't understand?
- What is the question or task asking you to answer?
- Is there enough information to find a solution?
- Restate the problem in your own words.
- What additional information do you need to find?

Develop a Plan

- There are many reasonable ways to solve a problem. With practice, students will build the necessary skills to choose an efficient strategy for the given problem.
- Ensure that students have a place to start and that the task/problem has the ability to be scaffolded.
- Caution should be exercised to not force your plan/reasoning on students.

Investigate

Productive Struggle

- Let students engage in productive struggle.
- Monitor as students work.
- Offer positive constructive feedback.
- Ask questions such as…
    - Why did you choose that number?
    - What assumptions did you make?
    - Explain what you are doing here.
    - What does that solution mean?

Questions for Individuals as they Work

If students have a hard time getting started…, then:

Ask students to describe the meaning of individual values in the table. For example, "What does the 181 mean"?

What is independence?

If students don't understand independence, then:

What is independence?

How do you test for independence?

What does it mean for two events to be independent? (use a Venn diagram & talk about the overlap - intersection)

Are you using all of the information from the question?

Did you find all of the probabilities?

What is your strategy to find each probability?

Could you identify each event (A and B) and their intersection?

Sample Solutions

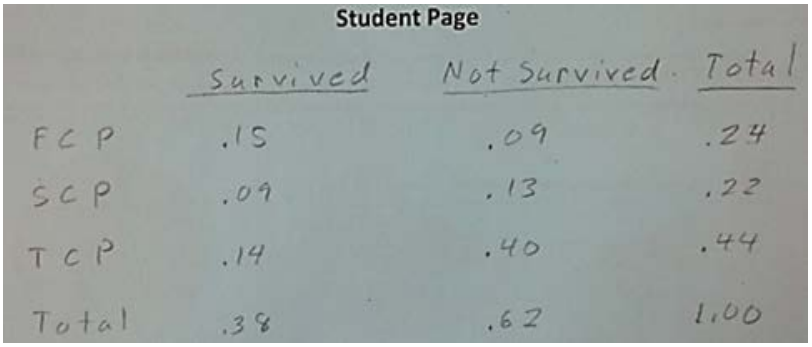
Incomplete work- student does not prove the events are independent.

Passenger survived: 500/1317, passenger in first class: 324/1317,

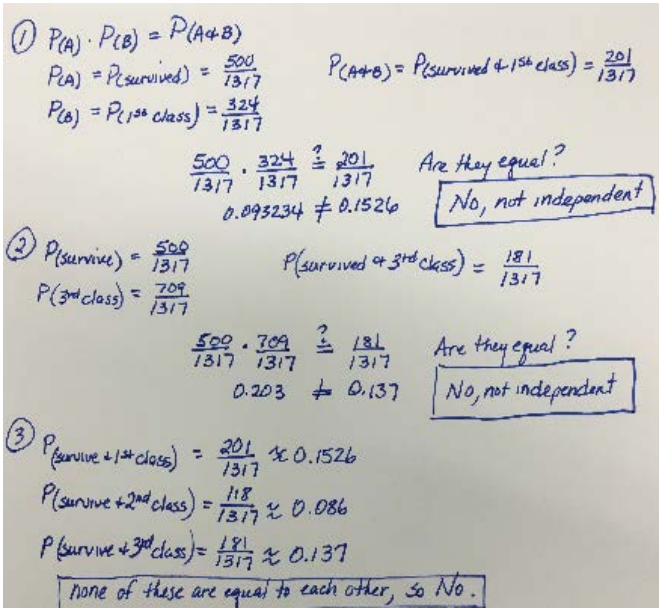
No, these are not independent events.

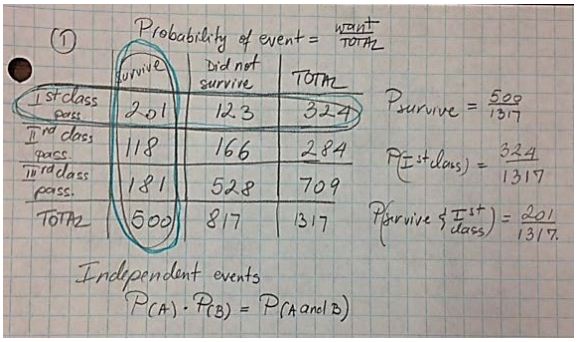
Passenger survived: 500/1317, passenger was in third class: 709/1317, No, these are not independent events.

First: 201/1317, Second: 118/1317, Third: 181/1317, No, all classes had different probabilities of surviving.


Student Page

	Survived	Not Survived	Total
FCP	.15	.09	.24
SCP	.09	.13	.22
TCP	.14	.40	.44
Total	.38	.62	1.00





Debrief

Whole/Large Group Discussion

- Debriefing formats may differ (e.g., whole-class discussion, small-group discussion). It will be beneficial for students to view student work as a gallery walk or similar activity.
- Have students/teacher facilitate the sequence of multiple representations in an order that moves from less to more mathematical sophistication.
- Allow students to question each other and explain their choices, using mathematical reasoning. If students struggle, use questioning strategies.
- Encourage students to notice similarities, differences, and generalizations across strategies.
- Provide constructive feedback and ask clarifying questions for deeper understanding of the process.

If you observe this …, you might ask this ….

Students talking about conditional probability… What words or symbols are frequently associated with conditional probability? How does conditional probability differ from straight probability?

Students unable to transfer the idea of independence… What is the idea of independence? Is it any more likely that you will get hit by a baseball tomorrow based on the fact that you didn't get hit by a baseball today? If no, then it is independent. If yes, then the events are not independent.

Looking at student work: What similarities do you notice in your answers? What differences do you notice in your answers? … Have students justify their answers to each other.

If you see this common error…, it might mean this…

Students using wrong values to calculate probability when doing conditional probability. Ask them to circle the whole column that represents event B. See if they understand that this is the condition that must be met or the "given" that has to occur before the event can happen. Have other students explain why this makes sense to them.

Synthesize and Apply

Monitor student work and facilitate discussions by asking questions. When students have independently arrived at the Enduring Understanding, engage them in solving these extension problems. Assess if you have facilitated the discussion in a way that students have arrived at the Enduring Understanding (do not tell them, they will benefit from discovering it for themselves).

Extension Problem #1

The following two-way table is comprised of data taken by the U.S. Census Bureau on the level of education reached by Americans of different ages. The total number of Americans in the table is 175,229.

Years of school completed, by age (thousands of persons)			
Education	Age group		
	25 to 34	35 to 54	55 and over
Did not complete high school	4,459	9,174	14,226
Completed high school	11,562	26,455	20,060
College, 1 to 3 years	10,693	22,647	11,125
College, 4 or more years	11,071	23,160	10,597

a) For Americans who are 35 to 54 years old, what is the probability of not completing high school?

b) What is the probability that someone was 55 and over and only completed high school?

c) How many Americans 25-34 year olds only completed 1 to 3 years of college? What's this probability?

Solution:

a) $\frac{9,174}{175,229}$ = 0.052

b) $\frac{20,060}{175,229}$ = 0.114

c) Number of people : 10,693;

Probability : $\frac{10,693}{175,229}$ = 0.061

Extension Problem #2

A large group of people was surveyed about their favorite movie genre. The participants had to give their age and choose their favorite genre from Action, Comedy, and Horror.

a) A company that sells a product designed for young adults is looking to advertise before the movies of one of these genres. Which genre should they choose? Explain your reasoning.

b) If you surveyed 12,000 people total, how many 18-25 year olds would you expect to choose Horror as their favorite genre?

c) If you surveyed 24,000 people total, how many 25-49 year olds would you expect to choose Comedy as their favorite genre?

	Action	Comedy	Horror
18-25 years old	238	450	312
25-49 years old	350	472	178
50+ years old	320	490	190

Solution:

a) Comedy, because 450 out of the 1412 movie goers choose comedy.

b) $\frac{12,000}{3,000}$ = 4 (the scale number) then, (4)(312) {312 = number of 18-25 y/o horror viewers} = 1,248

c) $\frac{24,000}{3,000}$ = 8 (the scale number) then, (8)(472) {472 = number of 25-49 y/o comedy viewers} = 3,776.

Extension Problem #3

Complete a two-way table and determine the requested probabilities.

48 students were asked to choose between horror films or romantic comedies. 21 likes only horror films while 15 liked both. 8 like neither.

a) P (romantic comedy):

b) P (did not like romantic comedy given that they liked horror):

c) P (like romantic comedy and like horror):

Solution:

	Horror	Non-horror	Total
Romantic comendy	15	19	34
Non-Romantic comedy	6	8	14
Total	21	27	48

a) $\frac{34}{48}$

b) $\frac{5}{21}$

c) $\frac{15}{48}$

References

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

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